VISCOELASTIC CHARACTERIZATION OF POLYMER-MODIFIED ASPHALT BINDERS OF PAVEMENT APPLICATIONS

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Abstract:

Rutting is a primary reason of premature deterioration of asphalt highway pavements. Pavements constructed with polymer and other modifiers are showing improved performance. The virgin asphalt and modified asphalt binders and mixes used on several test sections of the I-55 highway rehabilitation project in northern Mississippi are compared. The laboratory creep compliance data for these binders were measured at low temperatures using a modified test procedure adapted for the Bending Beam Rheometer device. Dynamic Shear Rheometer was used at high service temperatures. The creep compliance data of the binder was used as an input to simulate creep compliance behavior of the mix using a micromechanical model. The field evaluation confirms the relatively poor performance of the virgin asphalt section with respect to rutting, compared to modified binder sections.

ZUSAMMENFASSUNG:

Fahrrinnenbildung ist ein Hauptgrund für die vorzeitige Alterung von Asphaltbelägen auf Autobahnen. Beläge mit Polymeren und anderen Modifikatoren zeigen eine verbesserte Güte. Der unbehandelte Asphalt und modifizierte Zusatzstoffe und Mischungen, welche auf verschiedenen Testabschnitten des Sanierungsprojektes auf der I-55 im Norden Mississippis benutzt wurden, werden verglichen. Die im Labor ermittelten Daten der Kriechnachgiebigkeit für diese Zusatzstoffe wurden bei niedrigen Temperaturen gemessen, wobei eine modifiziertes Testverfahren benutzt wurde, welches an das Biegebalken Rheometer angepasst ist. Das dynamische Scherrheometer wurde bei hohen Temperaturen eingesetzt. Die Kriechnachgiebigkeitsdaten der Zusatzstoffe wurden als Eingabedaten zur Simulation des Kriechnachgiebigkeitsverhaltens der Mischung benutzt, wobei ein mikromechanisches Modell herangezogen wurde. Die Auswertung der Feldstudie belegt die relativ geringe Güte des Primärasphaltes in Bezug auf die Bildung von Fahrrinnen, verglichen mit den Abschnitten, welche mit modifiziertem Bindemittel asphaltiert wurden.

Résumé:

La première cause de détérioration prématurée des revêtements asphaltiques des autoroutes est la formation d'ornières. Les revêtements obtenus avec du polymère et d'autres adjuvants présentent des performances accrues. L'asphalte vierge, l'asphalte modifié avec des liants ainsi que des mélanges utilisés sur plusieurs sections tests de l'autoroute I-55 dans le cadre d'un projet de réhabilitation de cette autoroute du nord du Mississipi, ont été comparés. Des données de relaxation en fluage ont été obtenues en laboratoire pour ces mélanges à basses températures, à l'aide d'un rhéomètre de fluage dont le protocole expérimental a du être modifié. Un rhéomètre de cisaillement dynamique a été utilisé pour l'obtention de données à hautes températures. Les données de compliance en fluage obtenues pour l'asphalte modifié avec du liant ont été utilisées comme paramètres dans un modèle micromécanique pour simuler le comportement en fluage du mélange. L'évaluation faite sur le terrain confirme les performances relativement pauvres de l'asphalte vierge comparé à l'asphalte modifié avec du liant, en ce qui concerne la formation d'ornières.

Key Words: asphalt, binder, polymer-asphalt, laboratory, pavement, Superpave

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1 INTRODUCTION

Asphalt is predominantly used to construct pavements for roads, highways, and airports. Both asphalt binder and asphalt-aggregate mixture show temperature- and time-dependent behavior. Rutting or permanent deformation is the leading cause of pavement deterioration in temperate and warm climatic regions of the world and low-temperature cracking is a common problem in cold regions [1-4]. As the highway pavement infrastructure has aged and deteriorated, more pavements are in need of maintenance, rehabilitation, and reconstruction. This is traditionally done by placing hot-mix asphalt overlay. The Mississippi Department of Transportation (MDOT) has adopted the Superpave Performance Grade (PG) binder specification and mix design system [1], developed by the Strategic Highway Research Program (SHRP), that relates the asphalt mix design to performance. Polymer-modified asphalt has been claimed to resist rutting and has been used in a side-by-side experimental study in Mississippi.

This comprehensive study included laboratory testing for creep compliance characterization of asphalt layers, deflection testing, in situ pavement performance evaluation, and advanced finite element computer simulations of an in-service asphalt highway pavement on I-55 highway near Grenada in northern Mississippi [1-2]. The analysis and results of laboratory binder tests were conducted using Superpave equipment for different asphalt binders used in this study. The objectives of this paper are to: synthesize the laboratory test results of binders and mixes, compare the laboratory test results with the field performance in terms of resistance to rutting, and evaluate the effect of measured viscoelastic properties of the asphalt layer on finite element dynamic response analysis of the constructed asphalt pavement.

The pavement test sections were paved in 1996 with virgin asphalt and eight other modified binders including six polymers, two rubbers, and one chemical. Based upon the MDOT pavement design 76 mm thick (38 mm binder and 38 mm surface) asphalt layer was removed and replaced by the same thickness of new modified mix in each test section. The control and transition sections were constructed using AC-30 grade, the standard unmodified virgin asphalt used by the MDOT in Mississippi [1]. The optimum binder content of these mixes, determined using the Superpave level I mix design method, was 5.2 % by weight of total mix. Other details of the constructed sections are provided by Uddin and Yamini [2].

2 SUPERPAVE BINDER TESTS

The Superpave binder PG specifications and mix design are recommended by the Federal Highway Administration (FHWA) for improved and long life highway pavements [5, 6]. Laboratory Superpave binder tests were conducted at the University of Mississippi [7] using asphalt and modified asphalt binder samples from I-55 test sections. The PG binder for the test was PG58-10 (fast traffic), where 58-10 represents the high (58°C) and low (-10°C) pavement temperatures for northern Mississippi.

2.1 DYNAMIC SHEAR RHEOMETER TESTS

Dynamic Shear Rheometer (DSR) tests were conducted on all nine kinds of binders. In the DSR test operation, the asphalt sample is compressed between two parallel plates, one of which is fixed and the other one oscillates, as shown in Fig. 1. The DSR test measures the complex shear modulus, G^* , and phase angle, δ , of the binder. The complex shear modulus is a measure of total resistance of a material to deform when exposed to repeated pulses of shear stress. The phase angle is an indicator of the relative amounts of the recoverable and non-recoverable deformation.

The DSR tests were conducted at 46, 58, 70, 82°C for Novophalt and control asphalt samples. The tests were also conducted at several temperatures 46, 52, 58, 64, 70, 76, 82°C on Cryopolymer. For the rest of the asphalt binder samples the SHRP Spec Tests was conducted at 46°C



Figure 1: Dynamic Shear Rheometer (DSR) principle.



only because of time constraints. The DSR rutting factor, $G^*/\sin\delta$, represents a measure of the high temperature stiffness or rutting resistance of asphalt binders. The SHRP Superpave PG binder specifies minimum values for $G^*/\sin\delta$ of 1000 Pa for original asphalt binder and 2200 Pa after aging the binder using the Rolling Thin Film Oven (RTFO) procedure [7, 8].

The $G^*/\sin\delta$ results for all nine types of binders at 46°C are shown in Fig. 2. The results of $G^*/\sin\delta$ values were compared for control asphalt, Novophalt and Cryopolymer at 46, 58, 70, and 82°C. At 46 and 58°C Novophalt performs better than either control asphalt or Cryopolymer. At higher temperatures (70 and 82°C), all of the three binders perform in the same way as shown in Fig. 3.

The results in Fig. 2 and Fig. 3 show that Novophalt may offer better resistance to rutting at high service temperature. It is observed that, with the exception of Ultrapave, the modified asphalt binders and the control asphalt meet the SHRP PG specification requirements. This does not support the poor field performance of control asphalt, compared to the modified binders. Therefore, it can be concluded that the SHRP Spec Test, originally designed only for virgin asphalt binders may not be applicable to modified asphalt binders. The DSR test can be adopted to conduct creep compliance tests of these binders at high temperatures representing in-service conditions in warm climatic regions.

2.2 BENDING BEAM RHEOMETER TEST

The Bending Beam Rheometer (BBR) is used to accurately evaluate binder properties at low temperatures at which asphalt binders are too stiff to reliably measure rheological properties





using the parallel plate geometry of the DSR equipment. Used together, the DSR and the BBR tests provide rheological behavior of asphalt binders over a wide range of in-service temperatures in cold to warm climatic regions. Fig. 4 shows a close up of the BBR equipment. The BBR test measures creep stiffness value at the lowest in-service temperature, S(t) as designated by SHRP, which is indicative of the susceptibility to low-temperature cracking. To prevent this cracking, S(t) has a maximum limit of 300 MPa, as required by the Superpave PG binder specifications. Since low-temperature cracking occurs only after the pavement has been in-service for some time, this specification addresses these properties using binder aged in both the Rolling Thin Film Oven (RTFO) and Pressure Aging Vessel (PAV) devices.

The rate of change of binder stiffness with time is represented by the m-value, which is the slope of the log stiffness versus log time curve from the BBR test results, as shown in Fig. 5a. A high m-value is desired because as the temperature decreases and pavement contraction begins to occur, the binder will respond as a material that is less stiff. This decrease in stiffFigure 2 (left above): Comparison of G^* /sin δ values for asphalt and modified asphalt binder samples at 46 °C.

Figure 3 (left below): Comparison of G^* /sin δ values for control asphalt, Novophalt, and Cryopolymer binders at various temperatures.

Figure 4 (right above): Bending Beam Rheometer (BBR) equipment.

Figure 5 a) (right below): Rate of change of binder stiffness with time, represented by the m-value. Figure 5 b) (left above): The m-value results at -12°C.

> Figure 6 (left below): Comparison of measured S(t) values for asphalt binder samples at -12°C.

Figure 7 (right above): Binder relaxation modulus for control asphalt and modified binders at 1 second from BBR tests at -12°C.

> Figure 8 (right middle): Relaxation modulus for control asphalt binder.

Figure 9 (right below): Relaxation modulus for control asphalt and modified binders at -12°C.







ness leads to smaller tensile stresses in the binder and less chance for low-temperature cracking. A minimum m-value of 0.3 after 60 seconds is required by the Superpave PG binder specification [7]. As shown in Fig. 5b, all binders show higher than the required minimum m-value. The *S*(*t*) test results of all these binders satisfy the SHRP Superpave PG binder specifications [7].

Styrell

Kraton

Binder

Fig. 6 compares the *S*(*t*) test results at -12°C. Excessively high stiffness values, shown by some polymer-asphalt binders (Novophalt, Kraton, and Multigrade), indicate that the asphalt mix produced using these binders may be susceptible to low-temperature cracking, especially in the areas of cold climate. On the other hand, control asphalt and other modified asphalt binders including Rouse Rubber, Sealoflex, and Styrelf may perform better.

3 BINDER CREEP COMPLIANCE TESTS USING BBR

Additional laboratory tests were conducted using BBR to evaluate creep compliance of the selected binders for longer duration of 3600 seconds. The creep compliance, as a function of time, is defined as:

$$C(t) = \frac{\varepsilon(t)}{\sigma} \tag{1}$$

where $\epsilon(t)$ is the time-dependent strain under a constant stress, σ . The creep compliance is the reciprocal of the relaxation modulus, *E*.

A modified test procedure for the BBR equipment was adapted for asphalt and modi-



fied binders at different temperatures. The modified test was conducted by increasing the total test duration for a longer time of 3600 seconds. The BBRAN computer program was developed to analyze and summarize the output generated by the BBR machine.

Creep compliance and relaxation modulus values were calculated for several binders at a temperature of -12°C. Tests at temperatures of -18, -12 and o°C were also conducted during this study. It is apparent from this study that the binder creep compliance data at o°C may not be reliable using the BBR equipment. Therefore, future binder creep compliance tests will be conducted at o°C using the DSR equipment. Fig. 7 shows the binder relaxation modulus calculated from BBR tests at -12°C and at 1 second. The relaxation modulus value for Sealoflex modified binder was similar to that of the control asphalt binder, an indication of better performance at low temperatures.

Fig. 8 plot for control asphalt binder shows an increase in the relaxation modulus when the test temperature is decreased. Fig. 9 shows a comparison of the relaxation modulus for control asphalt and modified binders at -12°C



test temperature. Fig. 9 shows that the asphalt modulus attenuation is relatively more both in the short-term and long-term ranges at -12°C test temperature. This test temperature is well below the low-temperature regime in northern Mississippi. The binder creep compliance test results and elastic properties of aggregate were used to predict creep compliance of the compacted mix using the micromechanical analysis approach [9, 10], as discussed in the following section.

4 MICROMECHANICAL MODEL TO PREDICT MIX CREEP COMPLIANCE

The micromechanics approach for material modeling examines the interaction of the constituent materials directly on a microscopic scale [9 - 11]. The micromechanics approach used in this study is based upon the application of Aboudi's "method of cells" micromechanical analysis to predict viscoelastic response of resin matrix composites [9, 12]. The coupling of the method of cells with the time-stepping algorithm provides an accurate model for the analysis of composite materials displaying viscoelastic behavior.

The most attractive advantage of using the micromechanics approach, incorporated in the ASPHALT program [10], is that overall properties of the mix may be determined from the known properties of the constituents. The timedependent behavior of asphalt or modified binder, as a nonlinear viscoelastic component (binder creep compliance data and bulk modulus) is characterized by a time-stepping algorithm through the use of the hereditary integral form of the constitutive equations. The aggregate is characterized as a linearly elastic material by its Young's modulus and Poisson's ratio.

Fig. 10 shows mix relaxation modulus results predicted by the micromechanical analy-



Figure 10 (left): Mix relaxation modulus predictions from the ASPHALT program.

Figure 11 (right): Comparison of lab relaxation modulus results for virgin asphalt and modified asphalt mixes at 10°C.

sis for the control asphalt and several modified binders at -12 $^\circ C.$

5 TEMPERATURE CONTROLLED STRESS-STRAIN TESTS ON COMPACTED ASPHALT MIX

The stress-strain behavior of asphalt and modified asphalt highway mixes is dependent on time of loading and test temperature. For accurately modeling asphalt mixes it is important to recognize the temperature- and time-dependent viscoelastic material behavior that can be characterized in the laboratory by simple static creep tests [10]. The environmental controlled Asphalt Testing Machine (ATM) was used to conduct these stress-strain tests at different temperatures on compacted asphalt mix specimens produced using the U.S. Army Corps of Engineers Gyratory Testing Machine (GTM). The GTM was developed to produce laboratory test specimens by a kneading compaction process using an appropriate gyration angle to simulate the field wheel rolling condition [13]. The principle of gyratory compaction has been adopted in the SHRP Superpave asphalt mix design system. Several GTM compacted specimens were fabricated using the binders from the I-55 test site [1].

The ATM tests were conducted on the GTM specimens at temperature of 10 $^\circ C$ for the

control asphalt and several modified binders and at 25°C for the control asphalt. In this paper, only the results of the tests conducted at 10°C are presented for comparing the behavior of the control asphalt with the behavior of the modified binders. The test procedure consisted of an application of a constant load on the surface of the asphalt compacted mix specimen. This load was applied for 3600 seconds and then released to measure the permanent deformation of the asphalt specimen. The data acquisition equipment reads the displacement measured by two vertical Linear Variable Displacement Transducers (LVDTs).

Fig. 11 shows a comparison of the relaxation modulus calculated from the test data measured for the first 1800 seconds. The relaxation modulus values are normalized by the initial relaxation modulus calculated at 0.5 seconds for the virgin asphalt mix. These plots show the time-dependent properties and viscoelastic behavior of the mix specimens. The modified asphalt mixes perform better than the unmodified control asphalt mix. More rutting of the asphalt section after four years, measured in the field, confirms this finding.

6 COMPARISON OF FIELD EVALUATION WITH SUPERPAVE BINDER TEST RESULTS

Field rutting measurements were made on periodic basis by the MDOT staff for all nine types of asphalt binder sections on highway I-55. Fig. 12 is the box plot of the mean rut depth measured on the outside wheel path (OSWP) for the control asphalt and modified asphalt binders sections in November 1999. It shows that the largest amount of rutting (about 8 mm) occurred in the control asphalt section and the minimum rutting occurred in the Rouse Rubber section and fol-



lowed by the Sealoflex, Styrelf, Kraton and Novophalt polymer-modified asphalt binder sections.

The mean rut depth measured in the outside wheelpath in November 1999 has been correlated with the rheological properties of these binders. The $G^*/\sin\delta$ values for temperatures of 46, 76°C for original binders, 76°C for Rolling Thin Film Oven (RTFO) aged binders, and 22°C for Pressure Aging Vessel (PAV) have been considered. These results were obtained from the tests conducted at the University of Mississippi [7] and from the MDOT State Study 123 report [8]. From the plots of these properties and field rut depth measurements in 1999, it is observed that the highest correlation exists between the mean rut depth and the DSR rutting factor $G^*/\sin\delta$ at a temperature of 76°C for both original binders as well as the RTFO aged binders. The BBR binder test result at -12°C for Rouse Rubber shows relatively higher relaxation modulus compared to the control asphalt, Sealoflex, Kraton, and Novophalt binders.

7 BACKCALCULATION OF IN SITU MODULUS VALUES

In this study nondestructive evaluation (NDE) of the pavement sections was conducted by measuring surface deflection data using the falling weight deflectometer (FWD), operated by the Mississippi Department of Transportation. The deflection data were measured in fall of 1997 and 1998 at peak loads of around 4083 kgf (9000 lbf) and higher in the outer wheel path, about 1m (3 ft) from the pavement edge [1]. The FWD deflection data measured at the last drop height were normalized to 4083 kgf (9000 lbf) peak load for analysis.

A comparative study using several backcalculation programs was done [1]. Based on the results of this study, the PEDD modulus backcalculation program was selected because it ensures the uniqueness of backcalculated modulus results by predicting seed modulus values using in-built nonlinear deterministic equations. The I-55 pavement structure model consists of a 76 mm (3 in) asphalt overlay on top of a 190.5 mm (7.5 in) asphalt layer, a 305 mm (12 in) cement treated base (CTB) and lime treated subgrade (LTS) layer, and a semi-infinite subgrade layer. In situ effective modulus values were backcalculat-

Figure 12: Mean rut depth data measured in 1999.



ed by the PEDD program using the FWD data collected on 9th September 1998 at in situ test temperatures in the range of 21.7 to 26.1°C (71 to 79°F). The in situ modulus results for the overlay layer are summarized in Fig. 13.

The in situ Young's modulus values of the overlay backcalculated from the dynamic deflection data also show smaller modulus for the control asphalt compared to all other binders. The modulus values of Rouse Rubber and Sealoflex pavement layers are closer to the control asphalt modulus indicating a desirable behavior at the low test temperature for in-service pavements in northern Mississippi. However, the lower modulus value of the control asphalt layer at the test temperature of 26.1°C (79°F) makes it more susceptible to rutting during the warmer in-service temperatures, which may exceed 50°C (122°F) during hot summer days.

8 3D-FE SIMULATION USING VIS-COELASTIC MATERIAL PROPERTIES

Key limitations of most backcalculation programs are the assumptions of linear elastic, homogenous and isotropic material properties for the pavement layers and static loading. Three dimensional-finite element (3D-FE) analysis enables for the evaluation of the three-dimensional state of stress and strain in a continuum. Dynamic, static, and nonlinear analysis can be conducted using 3D-FE models [14].

In this study, the LS-DYNA finite element code [15] was used to analyze and verify the backcalculated modulus values from the PEDD backcalculation program shown in Fig. 13. The 3D-FE half model developed for the I-55 test section contained 7313 solid brick elements and 8528 nodes. The boundary conditions for this 3D-FE model were fixed at the bottom and roller supports were used on the sides.

Material type 6 in the LS-DYNA finite element program was used to simulate viscoelastic behavior. The viscoelastic properties for the LS-DYNA input file are: the bulk modulus (B), the short-term and long-term shear modulus (G_o and G_η), and the decay parameter, β . The element's shear modulus value G(t) and the bulk



modulus, *B*, are expressed by the following relationships [15]:

$$G(t) = G_{1} + (G_{0} - G_{1})e^{-\beta t}$$
(2)

$$B = \frac{E}{3(1-2\nu)} \tag{3}$$

where *E* is the Young's modulus and v is the Poisson's Ratio.

These properties were calculated from the ATM tests conducted at 10°C on the GTM compacted specimens. From the relaxation modulus plots G_0 , G_1 , and β were calculated. The G_0 value was calculated at the time of 0.5 seconds and G_1 at the time of 3000 seconds.

A comparison between the elastic analysis and the viscoelastic analysis was done for the control asphalt and Sealoflex pavements subjected to the FWD impulse loading. Detailed input data and results are not shown here for brevity. The Sealoflex modified binder plot shows larger deflections from viscoelastic analysis, compared to the control asphalt plot, as shown in Fig. 14. This implies smaller in situ Young's modulus for the modified binder pavement layer. These results indicate that correct properties, appropriate material models, and 3D-FE response analysis are needed for correct performance modeling of asphalt pavements.

9 CONCLUSIONS AND RECOMMENDA-TIONS

Several modified asphalt binder and control asphalt samples from I-55 highway overlay project site were tested using the Superpave DSR and BBR equipment to measure and analyze conFigure 13 (left): The PEDD backcalculated in situ Young's modulus values for the overlay.

Figure 14 (right): Comparison of FWD deflection time histories from 3D-FE elastic and viscoelastic dynamic analysis. trol asphalt and modified asphalt binder properties. Creep compliance and relaxation modulus data for different binders were measured using a modified BBR test procedure. The performance of the modified asphalt binders was better than the virgin asphalt. This was confirmed by more rutting measured on the control asphalt section after four years of traffic.

The mix creep compliance and relaxation modulus results predicted using the micromechanics approach also indicate poor performance of the control asphalt section compared to Rouse Rubber, Sealoflex, and other modified binder sections. The correct time- and temperature-dependent characterization of asphalt materials is imperative for advanced computer modeling and simulation. These improved laboratory material characterization and analysis efforts are needed for more accurate and meaningful structural response analysis of asphalt pavements and prediction of their performance for resistance to rutting.

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DISCLAIMER

The contents of this paper reflect the views of the author who is responsible for the facts, findings, and data presented herein.

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